Institut for Theoretical Physics
University Leipzig

Prof. Dr. B. Rosenow
A. Afanah , M. Staats

Advanced Statistical Physics - Problem set 14 - Bonus

Summer Terms 2022

Hand in: Hand in tasks marked with * to mailbox no. (43) inside ITP room 105b by Friday 15.07. at 9:15 am.

22. Long-range interactions *

2+2+2+2+3+3 Points

Consider the Landau-Ginzburg Hamiltonian

$$\beta\mathcal{H} = \int d^d x \left[\frac{t}{2}\vec{m}^2 + \frac{K_2}{2}(\nabla \vec{m})^2 + u\vec{m}^4 \right].$$

The long-range interactions between the spins can be described by adding a term

$$\int d^d x \int d^d y \; J(|\mathbf{x}-\mathbf{y}|)\vec{m}(\mathbf{x}) \cdot \vec{m}(\mathbf{y})$$

to the Landau-Ginzburg Hamiltonian.

(a) Show that for $J(r) \propto 1/r^{d+\sigma}$, the Hamiltonian can be written as

$$\beta\mathcal{H} = \int \frac{d^d q}{(2\pi)^d} \frac{t + K_2 q^2 + K_\sigma q^\sigma}{2} |\vec{m}(\mathbf{q})|^2 + u \int \frac{d^d q_1 d^d q_2 d^d q_3}{(2\pi)^{3d}} \vec{m}(\mathbf{q}_1)\cdot\vec{m}(\mathbf{q}_2)\vec{m}(\mathbf{q}_3)\cdot\vec{m}(-\mathbf{q}_1-\mathbf{q}_2-\mathbf{q}_3).$$

(b) For $u = 0$, construct the recursion relations for (t, K_2, K_σ). Find the fixed point corresponding to $K_2' = K_2$ and the anomalous dimensions y_t and y_{K_σ}. Similarly, find the fixed point corresponding to $K_\sigma' = K_\sigma$ and the corresponding anomalous dimensions y_t and y_{K_2}.

(c) Which of the fixed points controls the critical behavior of the system for $\sigma > 2$? How about in the case $\sigma < 2$? Which terms in the Hamiltonian are irrelevant?

(d) For $\sigma < 2$, calculate the generalized Gaussian exponents ν, η and γ from the recursion relations. Show that u is irrelevant, and hence the Gaussian results are valid, for $d > 2\sigma$.

(e) For $\sigma < 2$, consider $u \int d^d x \; \vec{m}^4$ as a perturbation, and use the perturbative RG (first order) to construct the recursion relations for (t, K_σ, u). Note that the calculation is analogous to the one discussed in the lectures.

(f) For $\sigma < 2$, it turns out that the recursion relations for t and u in the second order perturbative RG are modified to

$$\begin{aligned} \frac{dt}{dl} &= \sigma t + 4u\frac{(n+2)K_d\Lambda^d}{t + K_\sigma \Lambda^\sigma} - u^2 C_t \\ \frac{du}{dl} &= \epsilon u - 4u^2 \frac{(n+8)K_d\Lambda^d}{(t + K_\sigma\Lambda^\sigma)^2}, \end{aligned}$$

where $\epsilon = 2\sigma - d$. (In principle C_t could be determined by evaluating the diagrams appearing in the second-order RG calculation, but it is not necessary to know an expression for C_t.). Find the fixed points of the recursion relations. For $d < 2\sigma$, linearize the recursion relations in the vicinity of the non-trivial fixed point to find the critical exponents ν and η to first order in ϵ.

(g) What is the critical behavior if $J(r) \propto \exp(-r/a)$?